

Properties of Black Holes in Toroidally Compactified String Theory

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We review the macroscopic and microscopic properties of black holes of toroidally compactified heterotic and Type II string theory in dimensions $4 \leq D \leq 9$. General charged rotating black hole solutions are obtained by acting on a generating solution with classical duality symmetries. In $D = 4$, $D = 5$ and $6 \leq D \leq 9$, the generating solution for *both* toroidally compactified Type II and heterotic strings is specified by the ADM mass, $[\frac{D-1}{2}]$ -angular momentum components and five, three and two charges, respectively. We give the Bekenstein-Hawking entropy for these solutions, address the BPS-saturated limit and compare the results to calculations of the microscopic entropy both in the NS-NS sector and the R-R sector of the theory. We also interpret such black hole solutions as dimensionally reduced intersecting p-branes of M-theory.

1. Introduction

Black holes play an important role in string theory and recent developments (for a review, see *e.g.* [1,2]) have shown that string theory makes it possible to address their microscopic properties, in particular the statistical origin of black hole entropy, the radiation rates and possibly issues of information loss for black hole physics.

The starting point in such investigations is the classical black hole solution and one aim of this contribution is to review briefly the properties of a *general* class of these solutions in toroidally compactified heterotic and Type II string theories, *i.e.* for string vacua with enough (super)symmetries and thus well understood moduli spaces (This excludes notable examples of black hole solutions for $N = 2$ string vacua; see *e.g.*, Ref. [3] and references therein.).

In Section 2 we review the procedure to obtain the explicit form for general rotating black holes in dimensions $4 \leq D \leq 9$ and in Section 3 we concentrate on the structure of their Bekenstein-Hawking (BH) entropy. In Section 4 we review the progress made in identifying microscopic degrees of freedom of such black holes in order to obtain a microscopic (statistical) in-

terpretation of the BH entropy both for a class of black holes with charges arising in the Neveu-Schwarz- Neveu-Schwarz (NS-NS) as well as dramatic progress made in the Ramond-Ramond (R-R) sector of string theory. In Section 5 we briefly discuss the higher-dimensional interpretation of these black hole solutions as intersecting p-branes of the M-theory.

2. Classical Black Hole Solutions

The aim is to find general axi-symmetric solutions of the bosonic sector of the effective Lagrangian of toroidally compactified string theory. Since the bosonic sector includes the graviton, $U(1)$ gauge fields, as well as massless scalar fields, the axi-symmetric solutions correspond to the *dilatonic charged rotating black hole solutions*. Since for such configurations the scalar fields vary with the spatial direction, they affect their space-time structure and thus such solutions need not correspond to the solutions with regular horizons. It turns out, however, that when enough charges are turned on the scalar fields “slow-down” enough that the solutions effectively correspond to those with the global space-time of the Kerr-Newmann [Reissner-Nordström] black holes, *i.e.* global space-time is that of charged rotating [static] black holes of ordinary Maxwell-Einstein

*Compilation of talks given at SUSY'96, CERN Workshop on Duality in String Theory II, Strings'96 and Buckow'96.

gravity.

According to the “no-hair theorem” such black holes are specified by the ADM mass, $[\frac{D-1}{2}]$ -components of angular momentum and the number of allowed charge parameters associated with $U(1)$ gauge symmetry. *E.g.*, for toroidally compactified heterotic strings in $D = 4$, the general black hole solution is specified by the ADM mass, one component of the angular momentum, and 28 electric and 28 magnetic charges associated with 28 $U(1)$ gauge-fields.

Among such black hole solutions those which saturate the Bogomol’nyi-Gibbons-Hull bound [4], are special. They correspond to configurations with a number of preserved supersymmetries and for enough ($N \geq 4$ in $D = 4$) supersymmetries their classical properties are protected from quantum corrections. Thus we can trust their classical properties in the strong coupling regime as well. Such solutions play a crucial role [5] in establishing duality symmetries between different string vacua. At special points of moduli space, where they can become massless, they may enhance gauge symmetry [6,7] as well as supersymmetry [8,9]. When their space-time is that of black holes with regular horizons they turn out to play a crucial role as solutions whose microscopic entropy can be identified with the BH entropy.

2.1. General Solution from Generating Solution

The most general black hole, compatible with the “no-hair theorem”, is obtained by acting on the generating solution with classical U - (S -, T -) duality transformations (See Tables 1 and 2). These are symmetries of the supergravity equations of motion, and so generate new solutions from old. They do not change the D -dimensional Einstein-frame metric but do change the charges and scalar fields. One first considers transformations, belonging to the maximal compact subgroup C_U of duality transformations (See Tables 1 and 2), which preserve the canonical asymptotic values of the scalar fields and show that all charges are generated in this way. Another duality transformation can be used to change the asymptotic values of the scalar fields. Ultimately,

the solution could be cast in the manifestly duality invariant form.

2.1.1. Duality Symmetries

The low-energy effective action for the Type II string (or M-theory), toroidally compactified to D -dimensions, is the maximal supergravity theory, which has a continuous duality symmetry U of its equations of motion [10] (see Table 1, first column). U -duality has a maximal compact subgroup C_U (second column in Table 1). In the quantum theory the continuous classical symmetry is broken to the discrete subgroup Q_U [5] (third column in Table 1) of U -duality symmetry.

Toroidally compactified heterotic strings in D -dimensions inherit the classical symmetries of the even self-dual lattice of the (bosonic) gauge sector of the heterotic string and that of T^{10-D} torus (the first column in Table 2), referred to as continuous T - and S -duality symmetry [11]. The maximal compact subgroup C_U is denoted in the second column of Table 2. The conjectured quantum symmetry is the \mathbf{Z} valued subgroup of the classical symmetry (the third column of Table 2).

2.1.2. Solution Generating Technique

The general black hole solution is obtained by acting on a generating solution with the subset of U - (T -, S -) duality transformations. First, the asymptotic value \mathcal{M}_∞ of the scalar field metric \mathcal{M} can be brought to the canonical value $\mathcal{M}_{0\infty} = \mathbf{1}$ by a suitable U -duality transformation Ω_0 . The canonical value $\mathcal{M}_{0\infty}$ is preserved by C_U , *i.e.* the maximal compact subgroup of the U -duality group. The most general solution with the asymptotic behavior $\mathcal{M}_\infty = \mathcal{M}_{0\infty}$ is then obtained by acting on the generating solution with a subset of C_U transformations, *i.e.* the C_U orbits which are of the form C_U/C_0 where C_0 is the subgroup preserving the generating solution. In particular, with this procedure the complete set of charges is obtained. Indeed, with the generating solution labeled by n_0 charges ($n_0 = 5, 3, 2$ for $D = 4, 5, \geq 6$, respectively) and the dimension of the C_U orbits being n_1 , then $n_0 + n_1$ is the correct dimension of the vector space of charges for the general solution. Finally, black holes with arbi-

| D | Classical Duality- U | Maximal Compact Subgroup- C_U | Quantum Duality- Q_U |
|---|----------------------------------------------|---------------------------------|----------------------------------------------|
| 4 | $E_{7(7)}$ | $SU(8)$ | $E_{7(7)}(\mathbf{Z})$ |
| 5 | $E_{6(6)}$ | $USp(8)$ | $E_{6(6)}(\mathbf{Z})$ |
| 6 | $SO(5, 5)$ | $SO(5) \times SO(5)$ | $SO(5, 5; \mathbf{Z})$ |
| 7 | $SL(5, \mathbf{R})$ | $SO(5)$ | $SL(5, \mathbf{Z})$ |
| 8 | $SL(3, \mathbf{R}) \times SL(2, \mathbf{R})$ | $SO(3) \times U(1)$ | $SL(3, \mathbf{Z}) \times SL(2, \mathbf{Z})$ |
| 9 | $SL(2, \mathbf{R}) \times \mathbf{R}^+$ | $U(1)$ | $SL(2, \mathbf{Z})$ |

Table 1

The classical and (conjectured) quantum duality symmetries [5] for toroidally compactified Type II string in $4 \leq D \leq 9$.

| D | Classical Duality- U | Maximal Compact Subgroup- C_U | Quantum Duality- Q_U |
|---|-------------------------------------|---------------------------------|-------------------------------------------------|
| 4 | $O(6, 22) \times SL(2, \mathbf{R})$ | $O(6) \times O(22) \times U(1)$ | $O(6, 22; \mathbf{Z}) \times SL(2, \mathbf{Z})$ |
| 5 | $O(5, 21) \times SO(1, 1)$ | $O(5) \times O(21)$ | $O(5, 21; \mathbf{Z}) \times \mathbf{Z}_2$ |
| 6 | $O(4, 20) \times SO(1, 1)$ | $O(4) \times O(20)$ | $O(4, 20; \mathbf{Z}) \times \mathbf{Z}_2$ |
| 7 | $O(3, 19) \times SO(1, 1)$ | $O(3) \times O(19)$ | $O(3, 19; \mathbf{Z}) \times \mathbf{Z}_2$ |
| 8 | $O(2, 18) \times SO(1, 1)$ | $O(2) \times O(18)$ | $O(2, 18; \mathbf{Z}) \times \mathbf{Z}_2$ |
| 9 | $O(1, 17) \times SO(1, 1)$ | $O(17)$ | $O(1, 17; \mathbf{Z}) \times \mathbf{Z}_2$ |

Table 2

The classical and (conjectured) quantum duality symmetries of toroidally compactified heterotic string [11] in $4 \leq D \leq 9$.

trary asymptotic values of scalar fields \mathcal{M}_∞ can then be obtained from these by acting with Ω_0 .

2.1.3. Charge Assignments for the Generating Solution

In the following we shall list the charge assignments which were shown to be the charge assignments for the generating solutions for *both* toroidally compactified heterotic string [12] and Type II [13] string vacua. These charges are chosen to be associated with the NS-NS sector of the Type II string (or toroidal sector of heterotic string). Since these sectors are the *same* for both the heterotic and Type II string the explicit form of the generating solution in both cases remains the same.

- $D = 4$

Generating solutions are specified in terms of *five* charge parameters. It is convenient to label these charges in terms of magnetic $P_i^{(1,2)}$ and electric $Q_i^{(1,2)}$ charges associated with $U(1)$ gauge fields $A_{\mu i}^{(1,2)}$ of the Kaluza-Klein (momentum) sector, and the anti-symmetric tensor (winding) sector, respec-

tively. Here, i denotes the i -th compactified direction. The charge assignment of the generating solution is the following: $Q_1^{(1)}$, $Q_1^{(2)}$, $P_2^{(1)}$, $P_2^{(2)}$ and $q \equiv Q_2^{(1)} = -Q_2^{(2)}$. The generating solution then carries five charges associated with the first two compactified toroidal directions of the NS-NS sector.

- $D = 5$

The generating solution is specified by three (electric) charge parameters: $Q_1^{(1)}$, $Q_1^{(2)}$, and \tilde{Q} . Here \tilde{Q} is the electric charge of the gauge field, whose field strength is related to the field strength of the two-form field $B_{\mu\nu}$ by a duality transformation.

- $6 \leq D \leq 9$

The generating solution is parameterized by two electric charges: $Q_1^{(1)}$, $Q_1^{(2)}$.

As an example we shall illustrate the solution generating technique for the case of $D = 4$ toroidally compactified Type II string [13] (the case of $D = 4$ toroidally compactified heterotic string [12,14] is analogous). One starts with the

generating solution specified above by the five charge parameters. The group $C_U = SU(8)$ preserves the canonical asymptotic values of the scalar fields and only the subgroup $SO(4)_L \times SO(4)_R$ leaves the generating solution invariant. Then acting with $SU(8)$ gives orbits

$$SU(8)/[SO(4)_L \times SO(4)_R] \quad (1)$$

of dimension $63 - 6 - 6 = 51$. The $SU(8)$ action then induces 51 new charge parameters, which along with the original five parameters provide charge parameters for the general solution with 56 charges.

2.2. Explicit Form of the Generating Solution

In order to obtain the explicit form of the generating solution (which fully specifies the D -dimensional space-time) one employs the following solution generating technique. The D -dimensional stationary solutions of the theory are described by an effective $(D-1)$ -dimensional action, which is obtained by compactifying the D -dimensional action of the theory along the Killing direction, *i.e.* time direction. Thus, acting with symmetry transformations of the $(D-1)$ -dimensional effective action on a known stationary D -dimensional solution, one obtains *new* D -dimensional stationary solutions. In particular, by acting with a subset of such symmetry transformations on the D -dimensional Kerr solution (neutral, rotating solution), the charged rotating solutions are obtained.

Specifically, in the NS-NS sector of the theory the $(D-1)$ -dimensional action possesses $O(11-D, 11-D)$ non-compact symmetry. Acting with boosts $SO(1,1) \subset O(11-D, 11-D)$ [15] on the D -dimensional Kerr solution, specified by the mass m and $[\frac{D-1}{2}]$ angular momenta $l_1, \dots, [\frac{D-1}{2}]$. *E.g.*, the two $SO(1,1)$ boosts with boost parameters δ_1, δ_2 generate the electric charges $Q_1^{(1),(2)}$ of the Kaluza-Klein $U(1)$ gauge field and the two-form $U(1)$ gauge field, respectively. The solution obtained in that manner is specified by the ADM mass, *two* $U(1)$ charge parameters, and $[\frac{D-1}{2}]$ angular momenta $J_1, \dots, [\frac{D-1}{2}]$. In that manner the charged generating solution for black holes in $D \geq 6$ is obtained [16]. In $D = 4, 5$ the addi-

tional boost transformations in the NS-NS sector are needed (See Refs. [17,18] for more details.).

A program to obtain the explicit form of the generating solutions for general rotating black hole solutions is close to completion. Particular examples of solutions had been obtained in a number of papers (for a recent review and references, see [1]). The explicit expression for the generating solution has been obtained in $D = 5$ [18] and $D \geq 6$ [16,19], however, in $D = 4$ only the five charge static generating solution [17] (see also [20]) and the four charge rotating solutions [21] were obtained.

3. Bekenstein-Hawking Entropy

While the explicit form of the generating solution is complicated, the global space-time structure can be easily deduced. In addition, it turns out that in this case the Bekenstein-Hawking (BH) entropy S_{BH} can be cast in a relatively simple, suggestive form. Here, $S_{BH} = \frac{1}{4G_N} A$, where A is the surface area determined at the outer-horizon r_+ and G_N is Newton's constant (we follow Ref. [22] for the definition of the ADM mass, charges and angular momenta and a convention that the D -dimensional Newton's constant $G_N^D = (2\pi)^{D-4}/8$.).

3.1. $D = 4$

Here we quote results for the generating rotating solution with four charges $Q_1^{(1)}, Q_1^{(2)}, P_2^{(1)}, P_2^{(2)}$, only. The global space-time is that of Kerr-Newmann black hole and the BH entropy can be cast in the following suggestive form [21]:

$$S_{BH} = 16\pi[m^2(\prod_{i=1}^4 \cosh \delta_i + \prod_{i=1}^4 \sinh \delta_i) + \{m^4(\prod_{i=1}^4 \cosh \delta_i - \prod_{i=1}^4 \sinh \delta_i)^2 - J^2\}^{1/2}], \quad (2)$$

where m, l are the ADM mass and the angular momentum per unit mass of the Kerr solution and $\delta_{1,2,3,4}$ are the four boosts specifying the four charges. The ADM mass M , the four charges $Q_1^{(1),(2)}, P_2^{(1),(2)}$, and the angular momentum J , of the rotating charged solution are defined in

terms of m, l and the four boosts as, respectively:

$$\begin{aligned}
M &= 4m(\cosh^2\delta_1 + \cosh^2\delta_2 + \cosh^2\delta_3 + \cosh^2\delta_4) - 8m, \\
Q_1^{(1)} &= 4m\cosh\delta_1\sinh\delta_1, \quad Q_1^{(2)} = 4m\cosh\delta_2\sinh\delta_2, \\
P_2^{(1)} &= 4m\cosh\delta_3\sinh\delta_3, \quad P_2^{(2)} = 4m\cosh\delta_4\sinh\delta_4, \\
J &= 8lm(\cosh\delta_1\cosh\delta_2\cosh\delta_3\cosh\delta_4 \\
&\quad - \sinh\delta_1\sinh\delta_2\sinh\delta_3\sinh\delta_4). \quad (3)
\end{aligned}$$

In the (regular) BPS-saturated limit ($m \rightarrow 0, l \rightarrow 0$, while $me^{2\delta_{1,2,1,2}}$ are kept constant), the second term in (2) is zero and the BH entropy takes the form [12]:

$$S = 32\pi m^2 \prod_{i=1}^4 \cosh\delta_i = 2\pi[Q_1^{(1)}Q_1^{(2)}P_2^{(1)}P_2^{(2)}]^{1/2}, \quad (4)$$

Note that as long as all the four charges of the generating solution are non-zero the BH entropy (4) is *non-zero* and the black hole solution has the global space-time of the extreme Reissner-Nordström black hole.

Expression (4) has a straightforward generalization to the manifestly S - and T -duality invariant form[14]. Namely, when expressed in terms of 28 conserved quantized electric charge and 28 magnetic charge lattice vectors $\vec{\alpha}, \vec{\beta} \in \Lambda_{6,22}$ (of a toroidally compactified heterotic string), the surface area can be written as [14]

$$S = \pi[(\vec{\alpha}^T L \vec{\alpha})(\vec{\beta}^T L \vec{\beta}) - (\vec{\alpha}^T L \vec{\beta})^2]^{1/2}, \quad (5)$$

where L is the $O(6,22)$ invariant matrix. In the Type II case the result can be expressed in terms of the unique $E_{7(7)}$ quartic charge invariant [23, 13].

Both, (5) and the U -duality invariant form of the BH entropy are independent of asymptotic values of the moduli and the dilaton coupling, thus suggesting that it may have a microscopic interpretation (as first anticipated by Larsen and Wilczek [24]).

3.2. D=5

In this case the generating solution is specified by *three* charges. The global space-time is that of the five-dimensional Kerr-Newman solution. The BH entropy can be cast [21] in the following sug-

gestive form as a sum of the two terms:

$$\begin{aligned}
S_{BH} &= 4\pi \left\{ [2m^3 \left(\prod_{i=1}^3 \cosh\delta_i + \prod_{i=1}^3 \sinh\delta_i \right)^2 \right. \\
&\quad \left. - \frac{1}{16}(J_\phi - J_\psi)^2 \right]^{1/2} \\
&\quad + [2m^3 \left(\prod_{i=1}^3 \cosh\delta_i - \prod_{i=1}^3 \sinh\delta_i \right)^2 \\
&\quad \left. - \frac{1}{16}(J_\phi + J_\psi)^2 \right]^{1/2} \right\}. \quad (6)
\end{aligned}$$

The ADM mass, the physical charges and the two angular momentum components $J_{1,2}$ of the generating solution are related to the boost parameters $\delta_{1,2,3}$ as well as the ADM mass m and two angular momentum components $l_{1,2}$ of the five-dimensional Kerr solution in the following way:

$$\begin{aligned}
M &= 2m(\cosh^2\delta_1 + \cosh^2\delta_2 + \cosh^2\delta_3) - 3m, \\
Q_1^{(1)} &= 2m\cosh\delta_1\sinh\delta_1, \\
Q_1^{(2)} &= 2m\cosh\delta_2\sinh\delta_2, \\
Q &= 2m\cosh\delta_3\sinh\delta_3, \\
J_\phi &= 4m(l_1\cosh\delta_1\cosh\delta_2\cosh\delta_3 \\
&\quad - l_2\sinh\delta_1\sinh\delta_2\sinh\delta_3), \\
J_\psi &= 4m(l_2\cosh\delta_1\cosh\delta_2\cosh\delta_3 \\
&\quad - l_1\sinh\delta_1\sinh\delta_2\sinh\delta_3). \quad (7)
\end{aligned}$$

The regular BPS-saturated limit (of the generating solution) is obtained by taking $m \rightarrow 0$, while keeping the three charges $Q_1^{(1)}, Q_1^{(2)}$ and Q , as well as J_ϕ and J_ψ finite. The surface area of the horizon is of the form:

$$S_{BH} = 4\pi[Q_1^{(1)}Q_1^{(2)}Q - \frac{1}{4}J^2]^{1/2}. \quad (8)$$

Expression (8) has a straightforward generalization to the manifestly S - and T -duality invariant form[18], expressed in terms of 26 conserved quantized electric charge lattice vectors $\vec{\alpha} \in \Lambda_{5,21}$ (of toroidally compactified heterotic string) and one conserved quantized charge $\tilde{\beta}$ (associated with the gauge field related to the two-form field by a duality transformation). The surface area can be written as [18]

$$S = 4\pi[\frac{1}{2}\tilde{\beta}(\vec{\alpha}^T L \vec{\alpha}) - \frac{1}{4}J^2]^{1/2}. \quad (9)$$

In the Type II case the result can be expressed in terms of the unique $E_{6(6)}$ cubic charge invariant [25,13]. Again, in both the heterotic and Type

II string case, the five-dimensional BH entropy of the BPS-saturated solution does not depend on the value of moduli or the gauge couplings.

3.3. $5 < D < 10$

The generating solution has the global space-time structure of the D -dimensional Kerr black hole. With all the angular momenta turned on the BH entropy can be expressed in a compact form only for four- and five-dimensional black holes [16] and becomes progressively complicated as the dimensionality increases $D > 5$.

We shall therefore concentrate on the near BPS-saturated limit where the BH entropy can again be written in a compact form. The near-BPS-saturated black holes have regular horizons provided that the angular momentum parameters $l_{1, \dots, [\frac{D-1}{2}]}$ have a magnitude which is smaller than that of m . More precisely, in the near BPS-saturated limit one has to take m much smaller compared to e^{δ_i} , such that (when measured in units of α') $Q_1^{(1),(2)} \gg m = \mathcal{O}(1)$. In addition, the angular momenta are kept small compared to charges, so that $Q_1^{(1)} Q_1^{(2)} \gg J_{1, \dots, [\frac{D-1}{2}]}^2 \gg \sqrt{Q_1^{(1)} Q_1^{(2)}}$. The first inequality ensures the regular horizon, while the second inequality ensures that the contribution from angular momenta to the entropy is still non-negligible macroscopically. Now, the BH entropy can be cast in the following form[16]:

$$S_{BH} = 2\pi \left[\frac{4}{(D-3)^2} Q_1^{(1)} Q_1^{(2)} (2m)^{\frac{2}{D-3}} - \frac{2}{(D-3)} \sum_{i=1}^{[\frac{D-1}{2}]} J_i^2 \right]^{\frac{1}{2}}. \quad (10)$$

4. Microscopic Entropy

4.1. Black Hole Micro-states as Elementary String Excitations

The proposal to identify the microscopic degrees of electrically charged BPS-saturated black holes with those of elementary string states is due to Sen [15]. He proposed to identify the area of BPS-saturated electrically charged spherically symmetric solutions, evaluated at the *stretched*

horizon, with the degeneracy of BPS-saturated states of string theory (with the same charge assignment and mass as that of the BPS-saturated electrically charged static black hole). Since the stretched horizon is determined up to $\mathcal{O}(\sqrt{\alpha'})$, the identification of the two quantities agrees up to $\mathcal{O}(1)$, only.

For the *rotating* electrically charged solutions one is faced with the problem that the BPS-saturated solutions have (in general) a naked singularity. On the other hand, the area of rotating BPS-saturated black holes, evaluated at the stretched horizon (whose value is chosen to be independent of the angular momenta and electric charges), turns out to be *independent of the angular momenta*. This result is therefore not in accordance with the expectations that BH entropy should depend on the angular momenta, in order to be at least in qualitative agreement with the logarithm of the degeneracy of the corresponding BPS-saturated string states with non-zero angular momenta.

In another proposal [16], the area of the *near*-BPS-saturated electrically charged black holes, evaluated at their *true* horizon (10), should be identified with the degeneracy of states of the BPS-saturated elementary string excitations with the *same* quantum numbers (for charges, angular momenta-spins and the mass). Namely, the role of the stretched horizon of the BPS-saturated states is traded for the non-extremality parameter m of the near-BPS-saturated states. The degeneracy of these states $d_{\tilde{N}_L}$ can be calculated [16] and yields:

$$\begin{aligned} S_{stat} &\equiv \log d_{\tilde{N}_L} \sim 4\pi \sqrt{\tilde{N}_L} \\ &= 4\pi \left(N_L - \frac{1}{2} \sum_{i=1}^{[\frac{D-1}{2}]} J_i^2 \right)^{\frac{1}{2}}. \end{aligned} \quad (11)$$

In order to ensure the statistical nature of the entropy we need to maintain $N_L \gg \frac{1}{2} \sum_{i=1}^{[\frac{D-1}{2}]} J_i^2$, while still allowing for the statistically significant contribution from spins, *i.e.* $\sqrt{N_L} \sum_{i=1}^{[\frac{D-1}{2}]} J_i^2 \gg 1$.

Note that in this case the microscopic entropy (11) and the BH entropy (10) are in qualitative agreement (for $m = \mathcal{O}(1)$).

4.2. Black Hole Micro-states as Quantum Hair of Solitonic Strings

An earlier approach to calculate the microscopic entropy of four-dimensional BPS-saturated black holes with regular horizons was initiated in Ref. [24] and was further elaborated on in Refs. [14,26] and [27]. These approaches identify the microscopic black hole degrees of freedom as quantum hair [24] associated with particular small scale (marginal) perturbations [14,27] of string theory, which do not change the large scale properties of the black hole solutions. On the one hand, the magnetic charges of such dyonic black hole solutions ensure that the classical solutions have regular horizons (and thus α' corrections are under control), while on the other hand they effectively renormalize [24,14,26,27] the string tension of the underlying string theory. Within this approach the correct dependence of statistical entropy on charges (and angular momenta) is obtained. However, in order to achieve a precise numerical agreement with (4) and (8) additional assumptions on relevant microscopic degrees of freedom are needed [26] (see also Ref. [28] and references therein.).

4.3. Black Hole Micro-states as D-branes

Even though we chose to parameterize the generating solutions in terms of fields of the toroidally compactified heterotic string [or equivalently in terms of the NS-NS sector fields of the toroidally compactified Type IIA string], these generating solutions can be mapped, using string-string duality [or U -duality], onto configurations with R-R charges of a Type IIA string compactified on $K3 \times T^2$ [or R-R charges of a Type IIA string compactified on T^6]. Thus, they have a (higher-dimensional) interpretation in terms of the (intersecting) D -brane configurations and their microscopic degrees are those of the D -brane configuration (see Ref. [29] for further details on the physics of D -branes.).

The first agreement between such a calculation of the microscopic entropy and the BH one was obtained by Strominger and Vafa [30] in the case of static BPS-saturated black holes in $D = 5$, and was further generalized to (near)-BPS-saturated (rotating) black holes in $D = 5, 4$ (for a review

see [1,2] and references therein.). In spite of the fact that the classical black hole picture is valid only when the D-brane system corresponds to the strongly coupled string theory (and thus the perturbative calculation need not be valid in this regime) the numerical agreement between the the BH entropy and the perturbatively calculated result for the statistical entropy is none the less dramatic. It suggests that in the (near)-BPS limit the weak string coupling calculations can be reliably carried over to the strong coupling regime.

We would also like to emphasize that the BH entropy for a generating solution of *non-extreme* rotating black holes in $D = 4$ (2) and $D = 5$ (6) is cast in a suggestive form as a *sum of two* terms of the type $\sqrt{n_L}$ and $\sqrt{n_R}$, the latter one disappearing in the BPS-saturated limit, or extreme Kerr-Newman limit. This structure strongly suggests that even for black hole solutions (far away from the extreme limit) there may be an underlying microscopic description in terms of excitations of a *non-critical* string model (along the lines of a recent proposal of Horowitz and Polchinski[31]).

5. Black Holes as Intersecting M-branes

A unifying treatment of string-theory black hole properties may arise by identifying such black holes as (toroidally) compactified configurations of intersecting two-branes and five-branes of eleven-dimensional M-theory. A discussion of intersections of certain BPS-saturated M-branes along with a proposal for intersection rules was first given in [32]. A generalization to a number of different harmonic functions specifying intersecting BPS-saturated M-branes which led to a better understanding of these solutions and a construction of new intersecting p-brane solutions in $D \leq 11$ was presented in [33] (see also related work [34–42]). Specific configurations of that type reduce to the BPS-saturated black holes with regular horizons in $D = 5$ [33] and $D = 4$ [34] whose properties are determined by three and four charges (or harmonic functions), respectively.

One can further generalize such BPS-saturated intersecting M -branes to the case of *non-extreme static* intersecting M -branes [43] as well as *non-*

extreme rotating intersecting M -brane solutions [44]. Such configurations should be interpreted as *bound state* solutions of M -branes with a *common* non-extremality parameter and *common* rotational parameters associated with the transverse spatial directions of the M -brane configuration.

There exists a general algorithm for constructing the overall conformal factor and internal components of the eleven-dimensional metric for such configurations. The space-time describing the *internal* part of such (intersecting) configurations is specified *entirely* by “harmonic functions” for each constituent M -brane (associated with each charge source) and the “non-extremality functions” (associated with the Kerr mass), which are in general modified by functions that depend on the rotational parameters.

On the other hand, the transverse part of the configuration, which reflects the axial symmetry of the solution, involves charge sources as well as the rotational parameters in a more involved manner and cannot be simply written in terms of modified harmonic functions and non-extremality functions, only. However, in the case of static solutions the transverse part has a uniform structure of the form $f^{-1}(r)dr^2 + r^2 d\Omega_{D-1}^2$ with $d\Omega_{D-1}^2$ given by the infinitesimal length element of the unit $(D-2)$ -sphere S^{D-2} .

As an explicit example, we write down a structure for the intersection of two membranes and two five-branes ($2 \perp 2 \perp 5 \perp 5$), which becomes, after a dimensional reduction, a four-dimensional non-extreme black hole with four-charges. Such an intersecting M -brane solution has the following structure for the eleven-dimensional metric [43,44]:

$$\begin{aligned} ds_{11}^2 = & (T_1 T_2)^{-1/3} (F_1 F_2)^{-2/3} [-T_1 T_2 F_1 F_2 dt^2 \\ & + F_1 (T_1 dy_1^2 + T_2 dy_3^2) + F_2 (T_1 dy_2^2 + T_2 dy_4^2) \\ & + F_1 F_2 (dy_5^2 + dy_6^2 + dy_7^2) \\ & + f'^{-1} dr^2 + r^2 d\Omega_3^2] \end{aligned} \quad (12)$$

where the “modified” harmonic functions T_i and F_i are associated, respectively, with the electric charges $Q_i = 2m \cosh \delta_{ei} \sinh \delta_{ei}$ and the magnetic charges $P_i = 2m \cosh \delta_{pi} \sinh \delta_{pi}$, and the

non-extremality functions f, f' are given by [44]

$$\begin{aligned} T_i^{-1} & \equiv 1 + f_D \frac{2m \sinh^2 \delta_{ei}}{r}, \\ F_i^{-1} & \equiv 1 + f_D \frac{2m \sinh^2 \delta_{pi}}{r}, \quad i = 1, 2, \\ f & \equiv 1 - f_D \frac{2m}{r}, \\ f' & \equiv f_D \left(1 + \frac{l^2}{r^2} - \frac{2m}{r} \right), \end{aligned} \quad (13)$$

where $f_D^{-1} = 1 + \frac{l^2 \cos^2 \theta}{r^2}$ and l is the angular momentum of the neutral solution. Here, the expression for $d\Omega_3^2$, reduces in the case of static solution ($l = 0$) to the line element of S^2 (in the case of non-zero l the explicit expression for $d\Omega_3^2$ is not known). For the explicit solutions of one, two, and three intersecting M -branes see Ref. [43] (in the case of static solutions) and Ref. [44] (in the case of rotating solutions).

6. Conclusions

Over the last year dramatic progress in understanding classical and microscopic properties of black holes in string theory has been made. In this contribution we have addressed some aspects of this progress.

In particular, we have reviewed the structure of the Bekenstein-Hawking entropy of general rotating black hole solutions in toroidally compactified string theory (in diverse dimensions) and have emphasized connections of its suggestive structure to its microscopic origin (in the (near)-BPS limit) from the point of view of elementary string excitations, small-scale oscillations of an underlying higher-dimensional configuration, as well as D-brane configurations. Even more tantalizing is the fact that generating solutions for four- and five-dimensional *non-extreme* black holes still possess a suggestive form, indicating that even in this case there may be an underlying microscopic description in terms of excitations of *non-critical* string configurations.

The suggestive interpretation of these configurations as *non-extreme rotating M*-brane configurations may also turn out to provide a useful tool to address properties of the underlying black holes.

7. Acknowledgments

I would like to thank C. Hull, A. Tseytlin, and especially D. Youm for fruitful collaborations and discussions on topics presented in this contribution. The work is supported by the U.S. DOE Grant No. DOE-EY-76-02-3071, the NATO collaborative research grant CGR No. 940870 and the National Science Foundation Career Advancement Award No. PHY95-12732.

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